

Edward Kanterian: *Frege: A Guide for the Perplexed*. London/New York: Continuum, 2012. ISBN 978-0-8264-8764-3; \$24.95, £14.99 (paperback); 248 pages.

Gottlob Frege's *Begriffsschrift* founded modern logic. It also began Frege's reflections on the philosophical questions logic raises. The depth and interest of these early reflections are well drawn out by Edward Kanterian's illuminating study of the *Begriffsschrift* ('Concept-Script'). Most of Kanterian's book consists in a close and critical examination of the concept-script and the more questionable philosophical views Frege built around it. Kanterian's approach is fairly skeptical, especially of Frege's hope that logic would help to "break the power of the word over the philosophical mind" (*Begriffsschrift*, Preface, quoted at Kanterian, 42). In this, the present work has affinities with earlier research on Frege by G. P. Baker and P. M. S. Hacker, and generally with the thought of P. F. Strawson and the later Wittgenstein (Baker & Hacker 1984; Strawson 1952 and 2004). The book is, I think, aimed more at scholars and students of Frege or philosophy of logic & language than at complete beginners: it seeks to question widespread views and readings rather than to unperplex those reading Frege for the first time.

What is a concept-script? Kanterian starts with some history. Thinkers like Leibniz and Trendelenburg hoped for a perfect language which would reflect the structure of thought and reality, letting us describe things as they are without the distortion of ordinary speech. Leibniz also sought a *calculus ratiocinator* – a tool for calculating truth and falsity in everyday cases as we do in mathematics. Frege took up both these ideas. His concept-script was meant as a sort of perfect language, displaying thought's pure form, unifying the sciences and letting us easily test the validity of arguments.

This sounds like logic could replace ordinary language. Indeed, Frege is well known for seeing logic as a universal medium. But as Kanterian shows, Frege also has a countervailing tendency to see logic as a tool. It is to ordinary language (says Frege) as the microscope is to the eye: more precise but less flexible. On this view logic is dependent on ordinary language, and its use limited to specific purposes.

Logic is like language in another way. Frege doesn't think logical propositions are purely formal. They have content – they are general truths about the realm of all objects (see Goldfarb 2010). (There's an interesting question, little discussed by Frege, about how we could know these truths. Kanterian speculates that Frege believed in a kind of logical intuition, like Kantian intellectual intuition.) This is bound up with Frege's view of logical laws. He argued strongly against psychologism, the idea that logical laws describe human psychology. Logic tells us how we should think, not how we do think. Kanterian agrees that psychologism is false, but raises problems for Frege's view. Frege holds that logical laws are fundamentally descriptive. They're supposed to imply prescriptions, but it's not clear why any prescriptions follow. Furthermore, Frege's view seems to allow that there could be illogical thinking. Kanterian suggests that it might be better to say logic is constitutive of thought, so illogical 'thinking' is not thinking at all. We might worry, though, that on this view it's impossible to draw a false inference – or, at least, a false inference will not count as an act of thinking. And this seems to go against our ordinary notion of 'thinking'.

Kanterian goes on to suggest that Frege's anti-psychologism has implications for the relation between mind and brain and the nature of reason. Science consists of thoughts, which can be true or false. If we identify thoughts with brain processes, we have to say those processes are true or false. But as Frege says:

Standing by the river one often sees eddies in the water. Now would it not be absurd to claim that such an eddy of water was valid or true? And even if the dance of the atoms and molecules in my brain was a thousand times more spirited and frenzied than the dance of the gnats on a summer evening, would it not be just as absurd to assert that the dance was valid or true? (Quoted at Kanterian, 38)

If we can't assert this, we undermine the truth of the scientific arguments too. So, Kanterian says: "A full

account of human reason can't be given in purely materialist terms" (38).

The first chapter ends with a brief and useful explanation of the *Begriffsschrift* system: its basic logical signs, its axioms and its peculiar two-dimensional symbolic design.

The bulk of the book consists in an analysis of philosophical issues raised in the first 11 or so sections of *Begriffsschrift*, which is usefully read alongside. As Kanterian says, this part of the *Begriffsschrift* "contains, in a condensed form, a great wealth of philosophical topics, many of which Frege will pursue later in life as well" (50). The discussion covers many fundamental notions in philosophical logic, including: subject and predicate, negation, conditionality, the unity of the proposition, identity, logical analysis, definitions, function, generality and inference. I will only discuss a few topics here.

The judgment-stroke. All of the formulas in the *Begriffsschrift* start with the assertion sign: $|-$. The vertical part is the judgment-stroke; the horizontal is the content-stroke. It turns out to be surprisingly hard to say what exactly the judgment- and content-strokes mean. The judgment-stroke, Frege tells us, is supposed to express affirmation or assertion. The general idea, I think, is to dissociate force from content. Consider the fact that a sentence 'A' can occur asserted or unasserted (in 'if A then B'). The judgment-stroke is supposed to display assertive force, which is missing in the second kind of use.

Kanterian considers two ways of understanding this. First we might think the judgment-stroke *performs* – actually does – the asserting. The problem is that no sign could do that. Assertion is done by speakers, not the signs they use; and no sign could guarantee that it was being used seriously. As Wittgenstein said: "It is quite impossible for a proposition to state that it itself is true" (*Tractatus Logico-Philosophicus* 4.442). The second possible view of the judgment-stroke is that it has content, like 'is a fact' or 'is true' or 'is asserted'. But on this view $|-P$ would just be a statement: 'the circumstance that P is true' or 'it is asserted that P'. And this, like the statement 'P', could occur asserted or unasserted; the judgment-stroke turns out not to do anything to assert the content. So: "[the judgment-stroke] cannot perform the act of assertion, and if it merely reports it, it misses its logical point" (62). (Kanterian goes on to discuss further problems about interpreting $-P$: does P stand for a declarative sentence or a nominalization of one?¹)

We might try to hang on to a performative view of $|-$. A word like 'hereby' has a performative role in language: that doesn't mean the word 'hereby' performs anything, but rather that it is used to perform speech-acts, and that we can't explain its meaning in terms of its contribution to truth-conditions. The judgment-stroke might have a similar role.² Kanterian could respond that a symbol like this has no place in logic, which is only concerned with truth, not speech-acts. This makes sense if we think of logic as the pure structure of thought. But it's not so obvious if we think of concept-script (as Frege sometimes does) as a tool for writing down proofs in mathematics and science, where we would want to distinguish definitions, suppositions, assertions and so on.

Subject & predicate. Frege claimed that the "distinction of *subject* and *predicate* finds *no place* in my way of representing a judgment" (*Begriffsschrift*, s. 3). Subject and predicate were historically thought of as parts of a proposition's logical form. In Frege's view they are just rhetorical or grammatical categories, not logical ones: they don't affect "conceptual content". Two considerations support this. First, we can usually transform a sentence so that its subject becomes the grammatical predicate, and its predicate becomes the grammatical subject, while the sentence's inferential relations stay the same. Thus: 'I built this house' becomes 'this house was built by me'. Second, we can put any subject-predicate sentence into a form where the whole content is in the subject, and the predicate is 'is a fact'.³ So 'Aristotle died at Syracuse' becomes 'the death of Aristotle at

1 Kanterian's doubts here echo Wittgenstein: "But 'that such-and-such is the case' is *not* a sentence in our language – so far it is not a move in the language-game. And if I write, not 'It is asserted that...' but 'It is asserted: such-and-such is the case', the words 'It is asserted' simply become superfluous" (*Philosophical Investigations*, section 22).

2 I owe this point to Prach Panchakunathorn.

3 Frege says: "Our symbolic language is a language of this sort; the symbol $|-$ is the common predicate of all

Syracuse is a fact'. Again, it looks like the subject-predicate distinction has nothing to do with content, since content can always be nominalized in this way.

Kanterian disagrees with Frege's claim to have eliminated subject and predicate from a judgment's content. His argument is as follows. The general structure of concept-script judgments is $| - P$. Frege says the predicate of the whole judgment is 'is a fact'. It is predicated of a definite description, containing the judgment's content – e.g. 'the death of Aristotle at Syracuse'. Now note that not every definite description can intelligibly be followed by 'is a fact'. We can't say "the inventor of the bicycle is a fact". Why not? What condition does a definite description have to satisfy to express a judgeable content? "These expressions differ from unjudgeable contents precisely because they contain a verb/predicate or an expression derived from a verb/predicate" (72). This means that to give a rule for the correct use of Frege's universal predicate, 'is a fact', we must rely on the notion of subject and predicate as part of the content which is judged. (Kanterian's argument here goes back to Wittgenstein: "that which is true must already contain the verb" [*Tractatus Logico-Philosophicus* 4.063].)

I think Kanterian has shown that we must be able to talk about something like subject and predicate in the content of ordinary non-mathematical judgments. But Frege might deny that subject and predicate are the only thing that will do. Instead, Frege could say, a judgeable-content must contain a function and an argument. This response depends on Frege's claims that function and argument can replace subject and predicate, that concepts are really functions – and on whether Frege has a method for identifying the function and argument of a sentence without first identifying its subject and predicate.

Negation. Frege has a judgment-stroke which expresses the assertion or recognition of a truth. Does he have a denial-stroke, expressing the acknowledgement that a sentence is false? No. The only way to show the falsity of P in concept-script is to write: $| - \text{not-}P$. And this is "not just a point about concept-script, but one about the nature of negation" (79). Frege argued in his later essay "Negation" that there is no act of negating or denying to parallel the positive act of judging. To deny that P is just to judge that $\text{not-}P$; "we acknowledge the falsity of a thought by admitting the truth of its contradictory" (Geach & Black 1970, 131). Kanterian objects on this point. He notes that in ordinary language there do seem to be speech acts of denial: when we reply to a long assertion with 'No!' or 'Nonsense!'; when we metalinguistically negate an ungrammatical sentence; when we reject an order or demand. Frege might not care about ordinary language – but there are tensions within Frege's own view. If the sign $| -$ is a predicate, then why can't it be negated? Most important, I think, is the problem about empty names (88). Can't we deny "Vulcan is larger than Mercury" without asserting "Vulcan is *not* larger than Mercury"?

Identity. What does it mean to say $A=B$? Frege's "On Sense and Reference" starts with this problem: is identity "[a] relation between objects, or between names or signs of objects?" (Geach & Black 1970, *translations* 56). Frege went on to say that in the *Begriffsschrift* he had held the second (metalinguistic) view. Kanterian shows that "[Frege's] self-interpretation is misleading. [...] His focus, early and later, is throughout on content, on the objects of thought and the way we determine them" (110). Early and late, Frege recognized that names are associated with modes of determination of content, which explain how it can be significant to learn that $A=B$. In the *Begriffsschrift* he said: "the same content can be fully determined in different ways, and *that*, in a particular case, the *same* content actually is given by *two modes of determining* it, is the content of a *judgment*" (*Begriffsschrift*, s. 8, quoted at Kanterian, 112).

The later Frege reified modes of determination into Senses and made them part of the content of proper names. Kanterian resists this move and gives an excellent defense of the early account. While the content of 'A' may be determined differently from the content of 'B', these modes of determination are just "part of how the expression is initially introduced and used" (116) – not part of the content of the names. So ' $A=B$ ' and ' $A=A$ ' have the same content. Someone may object that given $F(A)$, we can derive $F(B)$ from ' $A=B$ ' but

not from 'A=A'. This seems to suggest that the two sentences have different entailments, and therefore different content. Kanterian responds that if $A=B$ then "it is not true that $F(B)$ is a different consequence from $F(A)$ " (117).

We may wonder how identity statements can tell us anything new. Kanterian says they "don't have a metalinguistic content, but they do have a metalinguistic role and consequences" (113). If we know about the way 'A' and 'B' were introduced, and learn that $A=B$, we can infer that the A-mode and the B-mode determine the same content. But once we learn that $A=B$, this changes our linguistic practice: identities affect the way we understand and use names. We "connect with each other uses of names previously seen as separate" (117). Someone who has connected the two names in this way can no longer assent to $F(A)$ but not $F(B)$, or see a difference between their contents.⁴ So they no longer have separate patterns of usage for 'A' and 'B'. On Frege's later view, by contrast, even someone who knows that $A=B$ should still see a difference in content between 'F(A)' and 'F(B)'. This is too rigid and inattentive to the changes in linguistic practice that follow from discovering an identity (118).

I think this is both a convincing reading of Frege and a valuable treatment of identity. Kanterian shows that Frege's early view can deal with the objections often thought to require Senses, and contains insights lost in the later work.

Definition, analysis, logicism. Frege treats definition in concept-script as abbreviation: we take a complex sign, which already has a meaning, and stipulate that some simple sign will mean the same. It's hard to see how this could have philosophical import. How can a definition tell us anything new? But Kanterian argues that "the general idea of definition plays a crucial role in [Frege's] overall logicist project and its ontological aspects" (122). In particular, informal definition – where we take a concept from ordinary language, especially the language of arithmetic, and give a concept-script equivalent – is not like abbreviation at all; and it is central to Frege's substantive claims. For example: what is a number? Frege's answer is to define 0, 1, 2 and so on. This is supposed to establish the ontological claim that numbers are really logical objects. So "[t]he informal definition of number in terms of extensions of concepts presented in *Foundations* is anything but a trivial abbreviation of an already existing expression" (125). For example, Frege's definition of zero is "the number which belongs to the concept 'not identical with itself'" (Frege 1884, 74). This was not a commonly used expression before Frege, but '0' was.

In fact, as Kanterian shows, Frege's use of definitions is bound up with deep metaphysical convictions. Here is his reconstruction of Frege's reasoning. Everything can be defined except what is most primitive (which can only be hinted at). The logical primitives form the basic structure of concept-script, referred to by its primitive signs. Now, our ordinary uses of numerals refer to logical objects, but in a hazy way: we don't clearly grasp the objects we're talking about. We want to clarify by defining arithmetical terms, but not just any definition will be good enough. (E.g. we could define '2' as 'the number of tennis players in a singles match', but that wouldn't tell us what numbers are.) As Kanterian puts it, we want to "define arithmetical signs in terms of what is epistemologically and ontologically most simple" (130). To clarify our arithmetical notions, then, we define them informally in terms of the logical primitives. Then we state concept-script equivalents and abbreviate them into new signs. It turns out that these newly defined signs capture arithmetic's true nature. Frege's definitions are ontologically important because they build the concepts and objects of arithmetic out of simple, indefinable parts.

This brings out the metaphysical picture underlying Frege's use of definitions. Insofar as we're skeptical of the idea of absolutely simple, indefinable, fundamental elements of reality, we have reason to doubt the picture and the claims founded on it. Kanterian aptly quotes Wittgenstein's *Philosophical Investigations*, section 47:

It makes no sense at all to speak absolutely of the 'simple parts of a chair'. [...] We use the word 'composite' (and

4 The idea here is similar to P. F. Strawson's account of informative identities (Strawson 2004, 43ff.).

therefore the word ‘simple’) in an enormous number of different and differently related ways. [...] To the philosophical question ‘Is the visual image of this tree composite, and what are its constituent parts?’ the correct answer is: ‘That depends on what you understand by “composite”.’ (And that, of course, is not an answer to, but a rejection of, the question.)

Function-argument analysis. Frege claimed he had broken from traditional grammar, replacing “the concepts *subject* and *predicate* by *argument* and *function*” (*Begriffsschrift*, Preface). So judgeable-contents are now to be analyzed into function and argument. Kanterian criticizes this, saying that there’s a tension between the method and the purpose it’s put to. Function-argument analysis can give us an infinity of different analyses for any proposition, depending on which sign we take to be replaceable; in this sense, the analysis “has nothing to do with the conceptual content; it concerns only our way of looking at it” (*Begriffsschrift*, s. 9 [translations from Geach & Black, *Translations*]). But function-argument analysis is supposed to reveal the logical form of propositions, and shouldn’t a proposition have only one logical form? The answer to the puzzle is that one analysis represents the true structure, while the other possible analyses don’t. E.g. “all whales are mammals” really has the form “for all X, if X is a whale then X is a mammal”. Now, how can we tell which analysis gives us the proposition’s true form?

Frege says: “in view of the contrast determinate–indeterminate [...] the whole proposition splits up into *function* and *argument* as regards its own content, not just as regards our way of looking at it” (*Begriffsschrift*, s. 9). Kanterian spends some time on this obscure passage. The idea is to compare two sentences, e.g.:

- (A) This bird has a heart.
- (B) Every bird has a heart.

We might think they have the same function, *has a heart*, with different arguments, *this bird* or *every bird*. But if we try to read (B) like (A), it makes no sense: *every bird* is not a being with a heart in the same sense as *this bird*. This contrast tells us something about the logical form of both sentences. Kanterian objects that this procedure relies on “external criteria of distinction between grammatical-ontological categories” (154). How do we know that ‘every bird’ and ‘this bird’ are categorically different? By knowing what can ordinarily be predicated of ‘this bird’. But our knowledge of this is not in terms of function-argument analysis but the prior subject-predicate structure of ordinary sentences.

Kanterian covers some of Frege’s later thought. I will just consider one argument which relates to my earlier discussion. In “Function and Concept”, Frege sets out the function-theoretic structure of concept-script: concepts, logical connectives and quantifiers are all functions. “[A] concept is a function whose value is always a truth-value”.⁵ The concept *is the President* takes various objects as inputs and gives as outputs the True or the False, depending on whether the objects fall under the concept.

Kanterian has an interesting objection based on contingent thoughts. Note that if two functions give us different values for the same argument, they are different functions. Now consider

- (O) Obama is the president.⁶

(O) is true, but might have been false if Obama hadn’t won an election. Obama is the argument in both cases, but we have different values; so we have to say there are two different functions. If (O) were false it would contain a different function, a different concept, and express a different thought. The problem is, first, that you can grasp an empirical proposition without knowing its truth-value. Second, if you do think (O), your

5 “Function and Concept”, in Geach & Black, *Translations* 30.

6 The example is mine.

thought's content doesn't depend on whether it is true or false. The basic issue, I think, is that a function is logically related to its set of argument-value pairs, but a concept can be contingently related to its extension. A Fregean might propose in response that concepts are functions from objects *and possible worlds* to truth-values. The function *is the president* would take two arguments: an object and a world. Kanterian worries that this sort of response makes sense cognitively unavailable – something “more comprehensive than what a human mind can grasp” (201). I'm not sure: in grasping an ordinary function like “ $x + 2$ ” we also manage to grasp something comprehensive, with an infinity of arguments and values. Still, I think Kanterian is right to worry that the possible-worlds version puts sense cognitively out of reach (though not because of its comprehensiveness). It seems to me that on the possible worlds view, if we don't know which possible world we're in, we don't fully know what we're saying when we utter (O). And of course we can't know which world we're in without knowing whether (O) is true.

Kanterian notes that this problem does not arise in the field Frege was mainly concerned with – the language of arithmetic – since arithmetical statements aren't contingent. “To speak of a function-theoretic account of the language of life [...] may just be itself a fiction” (216).

Kanterian's excellent book provides a full and critical account of Frege's early thought in the *Begriffsschrift*. Frege's short work proves to be rich in ideas (not all of them good, or mutually consistent) which presage and shed light on his later views. Along the way Kanterian's book comments on major interpretive debates – most importantly, it makes a good case (with Baker & Hacker, against Dummett) that Frege's early thought does rest on the mathematical idea of function (see Baker & Hacker 1984; Dummett 1984). Due to its detailed textual work and involvement with the secondary literature, I think the book will be of most interest to those who already have some exposure to Frege or the philosophy of logic & language.

The book raises questions that go beyond Frege's doctrines. If the concept-script symbolism is equivalent to modern logic (218), while its philosophical underpinning is flawed, what does this tell us about the relation between logical symbolism and philosophical logic? Many philosophers have thought (following Frege) that we should look to formal logic to understand ordinary language, and to logical analysis for solutions to philosophical problems. Kanterian doubts this project. He asks, finally: could it be “one task of philosophy to break the power of the mathematical sign over the philosophical mind?” (219)⁷

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References

- Baker, G. P. / Hacker, P. M. S. 1984. *Frege: Logical Excavations*. New York: Oxford UP.
- Dummett, M. 1984. An Unsuccessful Dig. *The Philosophical Quarterly* 34(136).
- Frege, G. 1884. *Foundations of Arithmetic*, Austin (trs.) 1980. Evanston: Northwestern UP. (Original: Frege, G. 1884. *Grundlagen der Arithmetik*, Breslau.)
- Geach, P. / Black, B. (eds.) 1970. *Translations from the Philosophical Writings of Gottlob Frege*. Oxford: Blackwell.
- Goldfarb, W. 2010. Frege's Conception of Logic. In: Potter, M. / Ricketts, T. (eds.). *The Cambridge Companion to Frege*. Cambridge: Cambridge UP.
- Strawson, P. F. 1952. *Introduction to Logical Theory*. London: Methuen.
- Strawson, P. F. 2004. *Subject and Predicate in Logic and Grammar*. Aldershot: Ashgate.

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