

Uwe Meixner · Albert Newen (Hrsg.)

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Ancient Philosophy
With a focus on the Master Argument

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Buchbesprechungen

Book Reviews

Mark Siebel: *Der Begriff der
Ableitbarkeit bei Bolzano*

Sankt Augustin: Academia Verlag 1997
(Beiträge zur Bolzano-Forschung Bd. 7)

It is generally agreed that Bolzano was the first to give a viable definition of logical consequence. But he deemed this concept “too obvious and important ... to have escaped entirely the attention of logicians.” Yet it seemed to him that “the nature of this relation has not always been correctly grasped, or, if comprehended, discussed with insufficient generality, or without a precise definition ...” (WL (*Wissenschaftslehre*) § 155, II p. 128). Bolzano’s comments on his predecessor’s theories are generally (except for Hegel) most kind. He amends them and attributes insight and understanding where less generous observers may see only confusion.

No doubt, earlier Bolzano scholars – I am thinking of H. Scholz and Y. Bar-Hillel – thought of themselves as similarly gracious when they took Bolzano to have anticipated a fair bit of classical logic – even if eight of 22 theorems in Bar-Hillel’s reconstruction are not provable in Bolzano, while three of Bar-Hillel’s are anti-theorems in WL (Bar-Hillel, *Bolzano’s Propositional Logic*; cf. also Scholz’s review of Bar-Hillel and his *Die Wissenschaftslehre Bolzano’s*). When they wrote, the language and logic of Principia Mathematica was widely touted as the “ideal” language, the one true logic and form of expression alone suited to overcome metaphysical confusion and even political friction. In this view, if an earlier logic did not anticipate the precepts of the classical, it was of “merely historical” significance. Whatever their merit as enthusiastic revivers of interest in Bolzano, Scholz and Bar-Hillel savaged his logic, which, in later terminology, falls in the class of deviant logics. A far more sensitive reconstruction was later given by Berg (*Bolzano’s Logic*).

The book before us is a sympathetic treatment of Bolzano’s concept of consequence and much of his logic. It is historically sensitive and reflects the more relaxed approach to alternative logic systems of the last few decades. Siebel begins by exploring Bolzano’s well-known claim that logical relations hold between abstract entities, i.e. propositions and representations “in themselves” (p. 15–45). The resolute anti-psychologism implied here has long been recognized and acclaimed: according to Bolzano, pure logic is not concerned with *judgments*, mental manifestations, but with their contents. Less appreciated is the equally important point that the objects of logical inquiry, the relata of logical relations, are not linguistic entities. For example, synonymous expressions, like ‘male goose’ and ‘gander’, stand for the same abstract representation. In a logic so conceived, problems of synonymy cannot be addressed. As Siebel

points out, in Bolzano's logic of variation (of which more presently) abstract *representations* are varied, not their linguistic expressions (p. 86).

He then patiently explores various difficulties in Bolzano's approach. Since propositions are *defined* as either true or false, and since they and their parts are the only entities in the abstract logical realm, the common distinction between ill- and well-formed formulas, between sense and nonsense, does not come under scrutiny. Bolzano does indeed hold that all propositions have a standard form, viz. "A has (or lacks) property b". But this is not a rule of well-formation, since it is never violated: the abstract realm contains no nonsensical strings of representations. Siebel points out that a strict and a liberal characterization of propositions is supported by the text. At one point, Bolzano says that the predicate must be the representation of a property [*Beschaffenheitsvorstellung*] (WL § 81, I p. 393), but elsewhere he suggests that it might be any representation whatever, or even a proposition (WL § 127, II p. 17). This would allow "God has Kant is a bachelor" to be a proposition which, since nonsense is not allowed, must be presumed false. Rightly, Siebel is puzzled (p. 33 ff.).

Further difficulties arise from Bolzano's claim that a proposition is false if its subject representation is "empty" [*gegenstandlos*], does not have a referent. This does not imply the falsehood of "Round squares do not exist." Rather, Bolzano holds, that the canonical form of this claim is "The representation 'round square' is empty", which is true. Another problem is not addressed, however. If 'B' is a universal term, and A non-empty, then "A's are B's" is true, but its contrapositive "non-B's are non-A's" is false – not a desirable consequence.

After the basics, Siebel discusses Bolzano's method of variation (p. 59–80). From a given proposition other propositions can be generated by replacing certain components. For example, let "The man Caius is mortal" be given. There is then a set of propositions which differ from this only in the element 'Caius', among them "The man Titus is mortal," "The man Sempronius is mortal," and so on. By attending to these sets, certain properties of propositions can be determined. In this case, the replacements resulting in non-empty subject terms generate a set of only true elements (WL § 147, II p. 78). In Bolzano's terms, the proposition is universally valid, or analytic, "with respect to 'Caius'."

Several authors have claimed that the replacements must be "of the same category" as the original. Siebel takes exception. In our example, every substitution either generates a true proposition or else, if we substitute 'triangle' or other terms from a different category, a proposition with empty subject term. Bolzano, at least in this example, seems to prefer a proviso to categorial adequacy: a proposition is analytic with respect to m if every substitution on m is true, *provided Gegenständlichkeit* is preserved (WL § 148, II p. 83). It would seem, however, that "Caius is mortal" in contrast to "The man Caius is mortal" is not analytic with respect to Caius unless, indeed, some sort of categorial restriction is placed on the substitutions.

Siebel now turns to Bolzano's famous definition of logical consequence:

Propositions M,N,O,... follow [*sind ableitbar*] from propositions A,B,C,D,... with

respect to the variable parts i, j, \dots if every set [*Inbegriff*] of representations whose substitution for i, j, \dots makes all of A, B, C, D, \dots true also makes all of M, N, O, \dots true (WL § 155, II p. 114).

He makes several important observations about this, noting that it is not meant to define a semiotic relation, but one that holds among propositions *an sich*. Attempts to recast Bolzano's definition for formulas, linguistic entities, lead to different results: problems of synonymy would have to be addressed, which, as noted, do not arise in the realm of the *an sich*. More important is the observation that Bolzano has defined a triadic relation (p. 87 ff.), a point missed by some earlier commentators. Further, since the variable elements are specifically noted, Bolzano can describe partly material consequences, like "All men are mortal, therefore Caius is mortal," which is valid if with respect to 'mortal'.

Bolzano's precise delineation of arguments has the inestimable merit that the so-called asymmetry thesis does not hold. In classical logic, arguments that instantiate some valid schema are valid, but it does not hold that arguments that instantiate an invalid schema are invalid, since all arguments have (some) invalid form. For example, $A \rightarrow B, B \rightarrow A \models A \rightarrow A$ instantiates 28 schemata, four of them classically valid, the rest invalid. Bolzano's logic of variation (if amended to apply to classical sentence logic) can discern only eight forms. The main point, though, is that we do not know *what argument is meant*, until it is disambiguated by stating, for example, that it is valid with respect to A, B, C and is thus identified as a hypothetical syllogism. The general result of these disambiguations is that in Bolzano's dispensation it holds that arguments that have valid form are valid, invalid form invalid (p. 121 ff.).

Siebel goes on to make a rather puzzling point. He claims that arguments are unambiguously described by the listing of variable elements, but that this listing does not determine whether an argument is formally or merely materially valid. For example, he thinks that "A, therefore possibly A" will be considered a material consequence if 'possibly' is a material element, and formal if it is part of the formal repertoire. My thought is that in Bolzano's view the status of modal operators must be an objective matter, even if difficult to determine, subject to controversy, and perhaps unknown until Aristotle discussed it in the *Prior Analytics*. But even if it were not, it would still be the case that the asymmetry thesis fails, whether the arguments are formal or material.

Material or enthymemic consequence is an interesting subject. Bolzano notes that for the assessment of such arguments information other than logical knowledge is often required (WL § 223, II p. 392). Kambartel (*Bolzano's Grundlegung*, p. XVIII f., cited p. 139) thinks that this knowledge will consist of "physical laws, definitions of general concepts, ethical maxims and other universal forms of validity" (*Geltungsformen*). Indeed, Bolzano seems to imply as much. But often the validity of a material consequence will ride on plain contingent facts. Consider "The Independence was commissioned in 1803. Therefore the Independence was not a ship of the line." This, as it turns out, is valid with respect to 'Independence,' not because of some universal form, but because no ship of the line was commissioned in that year.

A thorny issue is Bolzano's contention that the premisses of an argument must be consistent with each other with respect to the variables of the deduction. Buhl (*Ableitbarkeit*, p. 20ff.) and others have noted that this has several unpleasant consequences: (a) not every sentence follows from itself; (b) Bolzano's logic is not monotonic – one cannot add arbitrary further premisses to a valid argument and preserve validity; (c) it is not the case that if B follows from A, then the denial of A follows from the denial of B; (d) *reductio* arguments are not valid. The last, as Siebel points out, is a condition often violated by Bolzano himself (p. 109). I shall come back to some of this later.

In a chapter on the properties of the consequence relation (pp. 143–152), Siebel corrects earlier commentators, noting that consequence, being a triadic relation, cannot simply be described as asymmetrical, transitive, etc. Bolzano himself had seen (though he did not have the technical vocabulary) that interesting properties obtain under certain circumstances. For example, if C follows from B, and B from A *with respect to the same set of variable terms*, then C follows from A with respect to that set. The relation is transitive in the first two positions if the third element is identical.

Now follows a perceptive comparison between Bolzano's notion of an argument form and Russell's propositional functions (p. 153–183). It had been claimed by several commentators, among them Buhl, Bar-Hillel and Scholz, that Bolzano anticipated Russell's notions, or had a crude equivalent of it in his method of variation. In criticizing this, Siebel makes the correct observation that Bolzano does not countenance variables in the modern sense, and that the *an sich* realm contains only *propositions* and their parts, not items with variables (p. 164). He further notes that in Bolzano's system the evaluation of an argument must take into account not only the functions that are instantiated, but also the truth value of the propositions of the argument itself – perhaps another untoward consequence of the consistency requirement (p. 168). Siebel does not note a formal point of considerable interest. A proposition f_{aa} can be an instance of two functions, viz. f_{xy} and f_{xx} . By variation of a in f_{aa} , however, we can obtain only a set of propositions with identical arguments, corresponding to f_{xx} , but not the set corresponding to f_{xy} . This is analogous to the hypothetical syllogism considered above, which instantiates 28 classical schemata, but is one of only eight forms in Bolzano's scheme. In relation to a given sentence or argument there are always more propositional functions than there are Bolzano – type forms.

Bolzano did in fact think that the form of propositions and arguments (relative to a set of "variable" terms) is a species or set (*Gattung*, WL § 12, I p. 48). He supports this with a whimsical reference to Cicero (WL § 81, I p. 391), who said that *forma* and *species* mean the same thing: "*utroque verbo idem significatur*" (*Topica* 30). I said in an earlier piece that such a set must be "generated" from a given argument, and incautiously added that "it is not a pre-existing thing in which the argument participates, but must be operationally developed from it" (*Bolzano's Consequence*, p. 306). Siebel points out that propositions are abstract objects that cannot "be generated." In response, I hereby withdraw the second incautious claim. This does not mean, however, that the vocabulary of

“generation” can easily be removed from the discussion of abstract objects. In the *Pure Theory of Numbers*, Bolzano says in the very first paragraph that he will call a member of a certain sequence a number, if it is represented by a term “that indicates its method of generation” [*ihre Entstehungsart*] (*Reine Zahlenlehre* p. 15). In a letter to Exner (Briefwechsel p. 83) he reflects on a “formula that generates all prime numbers.” Now if we take Siebel seriously, there can be no such formula, because no formula can “generate” abstract objects such as numbers (or anything else, for that matter). I prefer to continue to use this figure of speech, which cannot easily be replaced in mathematical discourse. In this case, we must insist on the distinction between “generating” a proposition or argument from a schema, and “generating” a form or species of propositions or arguments from a given proposition or argument.

There follows an important chapter comparing Bolzano’s concept of consequence with that of Tarski (p. 185–223). The difference between the definitions concerning the objects in question (propositions vs. formulas), the model theoretical vs. substitutional approach, Bolzano’s restriction to consistent premisses etc. are duly noted. It is a fine critical chapter that does not simply repeat earlier claims that Bolzano, with the limited means he had in hand, roughly anticipated Tarski’s definitive account.

Finally, Siebel discusses the issue of “relevance” (p. 225–256). Anderson and Belnap, and others have claimed that it is a necessary condition for the relevance of a premiss set to the conclusion of an argument that they share some element. In this sense the premiss ‘ $A \ \& \ \neg A$ ’ is not “relevant” to the arbitrary conclusion B , which it classically entails.

Bolzano did indeed state that in a valid argument, premisses and conclusion share a variable element (WL §155, II p. 120), but he does not give a good argument for this claim. I do not think that we can satisfactorily resolve this issue if we seek a proof in the text of WL. There is too much uncertainty, for example, whether we should take the consequence relation to be contrapositive. If we do, and if we also stipulate that the conclusion not be analytic with respect to the variables of the deduction, then a plausible argument for relevance can be mounted. But this means that we approach Bolzano’s theory in a spirit of correcting, amending and enlarging, in the spirit in which, for instance, Euclid’s *Elements* have been treated over the centuries. The *Elements* suffer from various deficiencies, lacking, for instance, a satisfactory definition of similarity, which is defined only *ad hoc* for triangles, regular polygons and polyhedra, but not for other figures and bodies. In general, if a formal system has enough structure and content, its deficiencies can with good conscience be corrected in appropriate ways. We can, and I think should, do this with Bolzano, for instance by extending his triadic conception of the consequence relation to formulas with well defined grammars, such as propositional formulas with classical connectives.

The systems that result may, in various ways, contravene Bolzano’s explicit stipulation. For example, we may wish to insist on the validity of $A \ \& \ \neg A \models A$, despite Bolzano’s explicit denial (WL §155, II p. 115). This is because the payoff, in propositional logic, of construing consequence as triadic, depends

on the possibility of varying compound expressions independently of their constituents. If we don't do this, then a Bolzano version of propositional logic collapses into an uninteresting fragment of the classical. On the other hand, if, in this case, we vary $\neg A$ independently of A , the premisses are in fact consistent *with respect to the variable elements*, and in this sense the argument satisfies the consistency requirement. I have argued elsewhere that this liberal form of substitution still does not allow $A \& \neg A \vDash B$. If we add the condition that the conclusion not be analytic with respect to the variable elements, we also prevent $A \vDash B \vee \neg B$ and, in fact, have described a relevant consequence relation (*Bolzano's Consequence*, p. 310f.).

Siebel chose not to take such a radical approach. Instead, he gives a sensitive and accurate exposition of Bolzano's stated doctrine, with due attention to its various problems. More than that, his several comparisons with contemporary and recent theories – Russell on propositional functions, Wittgenstein on material consequence, Tarski on logical consequence, relevance logic and various others – are invaluable. They help to clarify, through comparison and contrast, many of Bolzano's tenets. This is a very important study in a most valuable series.

Rolf George, University of Waterloo, Canada

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